Concept of Four Basic Operations:

"Operations on natural numbers usually form a major part of primary mathematics syllabi. However, the standard algorithms of addition, subtraction, multiplication and division of whole numbers in the curriculum have tended to occupy a dominant role in these. This tends to happen at the expense of development of number sense and skills of estimation and approximation. The result frequently is that students, when faced with word problems, ask "Should I add or subtract? Should I multiply or divide?" This lack of a conceptual base continues to haunt the child in later classes. All this strongly suggests that operations should be introduced contextually. This should be followed by the development of language and symbolic notation, with the standard algorithms coming at the end rather than the beginning of the treatment".

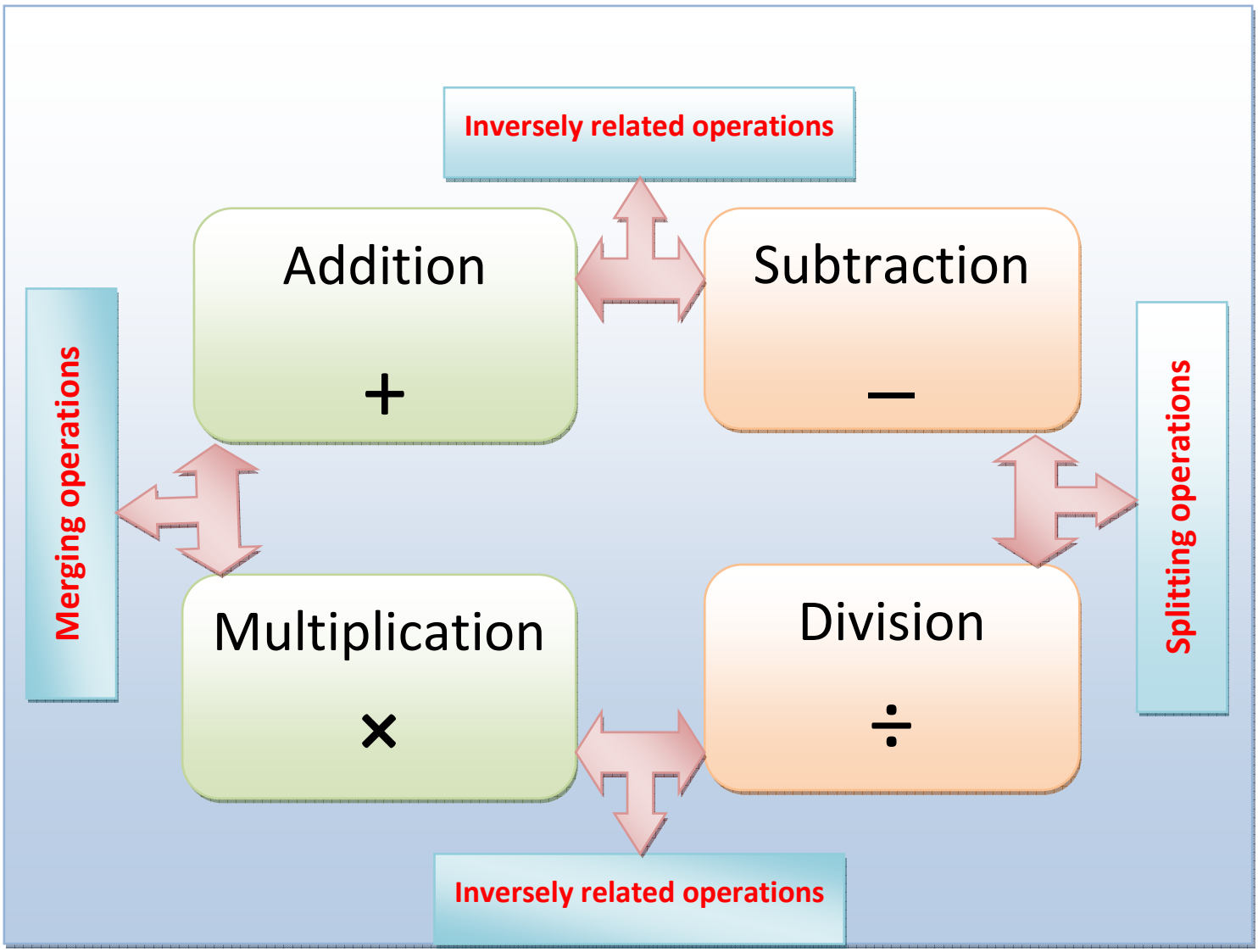
This paragraph has been taken from the 'Position Paper' for National Curriculum Framework 2005. The important takeoff from this statement is – before going to standard algorithm, basic operations should be introduced contextually and with lots of langue inputs. While working with basic operations we have to keep in mind two major objectives – the conceptual understanding and procedural fluency.

- **Conceptual Understanding:** In the conceptual understanding we should give emphasis on fundamental concept behind each operation.
- **Procedural Fluency:** Procedural fluency refers to the ability to perform procedures flexibly, accurately and efficiently.

Importance of Basic Operations

The four basic operations are the basics of Mathematics. Addition, subtraction, multiplication and division are the fundamentals of mathematics, which are also used in everyday life. All the equations, graphs and an enormous amount of other significant calculations, in one way or another, can be broken into these four basic operations of mathematics. If we consider mathematics as a machine, then addition, subtraction, multiplication and division are its tools, performed on numbers or values. The four basic mathematical operations- addition, subtraction, multiplication and division have application even in the most advanced mathematical theories. Thus, mastering them is one the keys to progressing in an understanding of advance mathematics and especially of algebra.

Four basic operations and their relation:



Addition and Subtraction:

Addition and its inversely related operation, subtraction are powerful foundational concepts in mathematics, with applications to many problem situations and connections to many other topics. Addition determines the whole in terms of the parts and subtraction determines a missing part. We can actually define subtraction in terms of addition.

Although addition is the opposite of subtraction, it is also true that every addition problem can be rewritten as a subtraction problem. For example, the problem 3 + 2 = 5 can be rewritten as the subtraction problem 5 - 3 = 2 or 5 - 2 = 3. We may introduce this with language problem, as example you have 5 balloons and you gave 3 balloons to your brother. Now how many balloons have left with you? Or you have 5 Laddus you gave 2 Laddus to you friend, how many Laddus are now with you? Or you had 5 chocolates with you and you gave a few to your sister. Now you have 2 chocolates with you. So how many you gave to your sister? It should be looked that, the sum 5 in the addition problem became the minuend and the other numbers became the subtrahend and the difference.

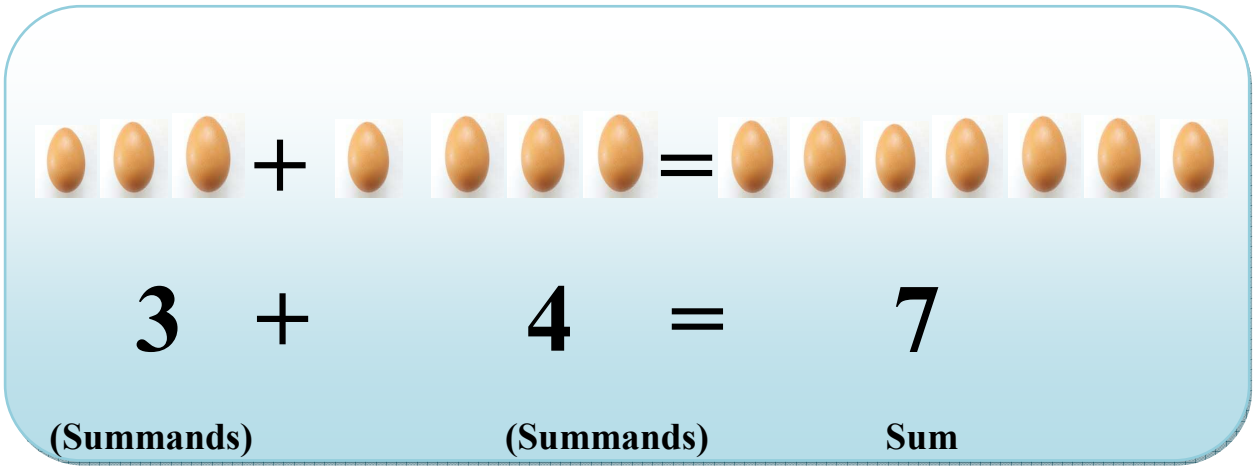
Let us take another example the relation between these two operations: **Rumi went to a shop with 100 rupees and took material of 65 rupees. Now how much amount is left to Rumi?**

If we ask, how this problem can be solved in school mathematics session? The most common answer will be this is a problem of subtraction. But in real life situation a shopkeeper solve this problem by adding amount with 65 and reach to hundred.

Concept of Addition:

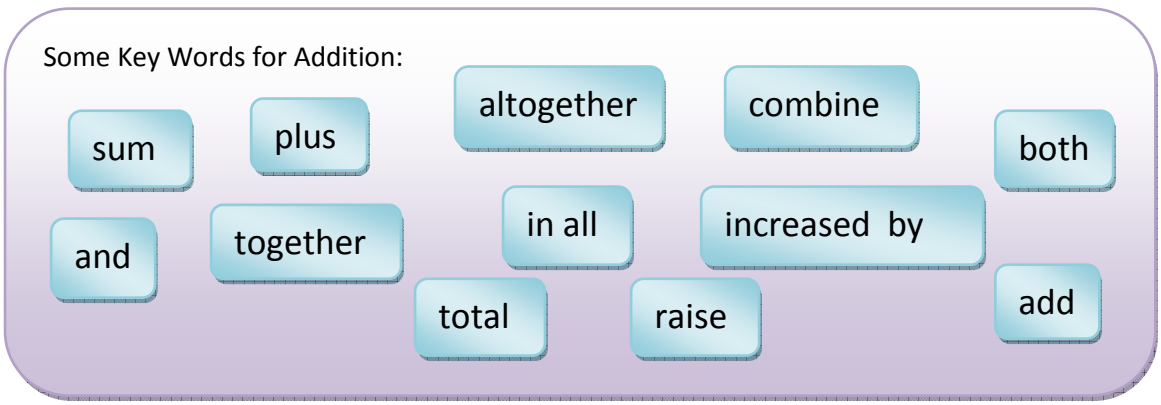
Addition is simply the combination of distinct sets of like entities. Thus, if we add one set of 3 eggs with another set of 4 eggs, we get a total of seven (7) eggs.

The above example is an illustration of the process of addition. Please note here that the plus sign (**+**) indicates that the operations performed on the two **terms.** In this case, Summands are 3 eggs and 4 eggs. The equal sign (**=**) indicates that what is on its left and what is on its right are equivalent (or equal). On the right side is the **Sum,** which is the result of the addition of the summands.



Language and Addition:

It is very important that students see mathematics, and the calculations they perform, as part of their daily life. Providing opportunities to apply basic concepts and operations in daily activities will reinforce students' skills and motivate them to progress in mathematics. They can use addition to figure total amounts of toys or chocolates, and to keep track of their study materials or play equipments for their team. Teacher should motivate students to construct their own language problems of addition. As an example I have 5(five) marbles earlier now I got another 4(four) from my brother, so now I have all together 9 (Nine) marbles, or 5 + 4 = 9. Always initial concept should be given with lots of language inputs.



Properties of Addition:

1. Commutative Property

This property states that numbers can be added together in any order without changing the resulting sum.

Example: 5 + 3 = 8, 3 + 5 = 8 or a + b = b + a

2. Associative Property

According to associative property of addition, the sum of three or more numbers remains the same regardless of how the numbers are grouped.

Example: (4 + 3) + 1 = 4 + (3 + 1) = 8, or (a + b) + c = a + (b + c)

3. Additive Identity Property

Additive Identity Property means, any number plus zero is the original number.

Example: 3 + 0 = 3, ½ + 0 = ½, or a + 0 = a

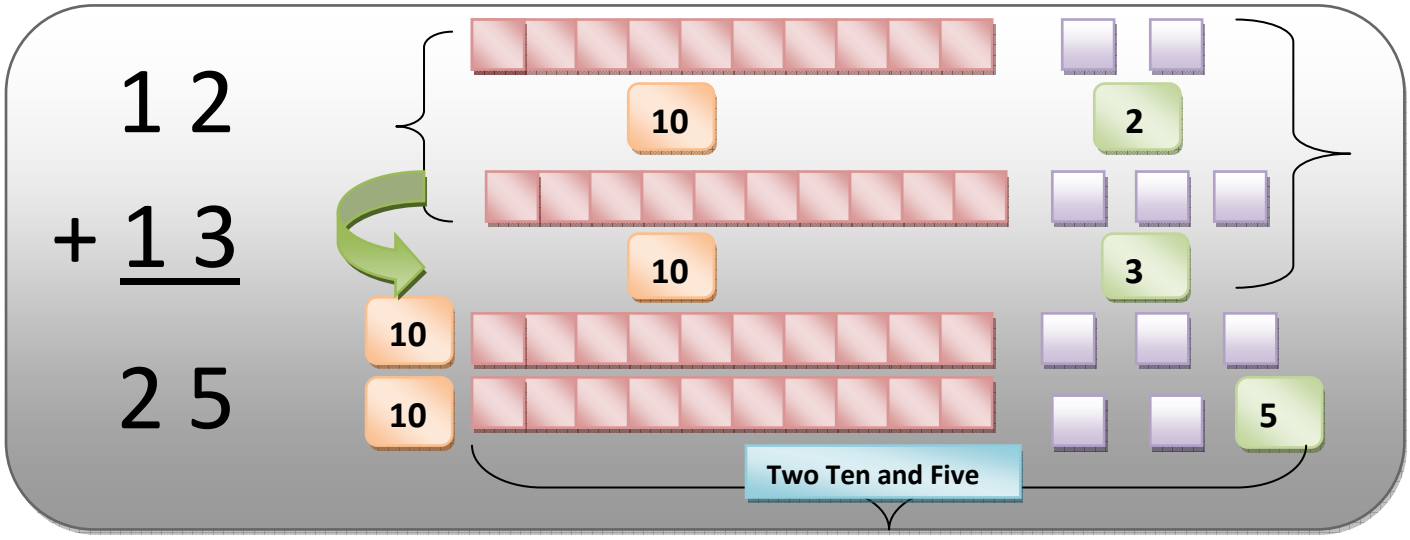
4. Distributive Property

The distributive property is easy to remember, if you recall "multiplication distributes over addition".

Example: 2 (3 + 4) = 2 x 3 + 2 x 4, or a (b + c) = ab + bc

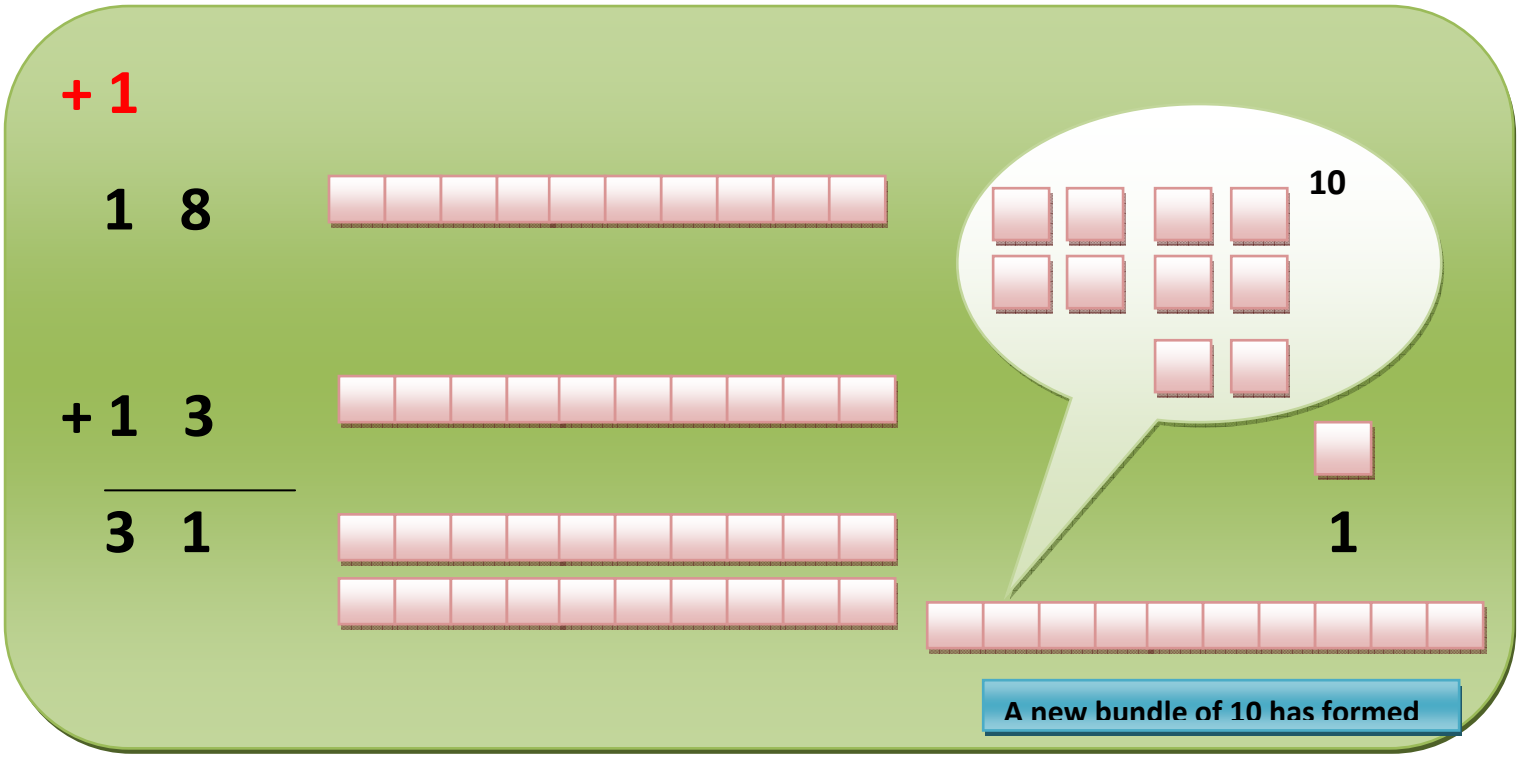
Concept Building activity for double digit addition:

The teacher will use the concept of 'bundles' and that can be done with concrete materials and after that with picture. Let us take an example here -



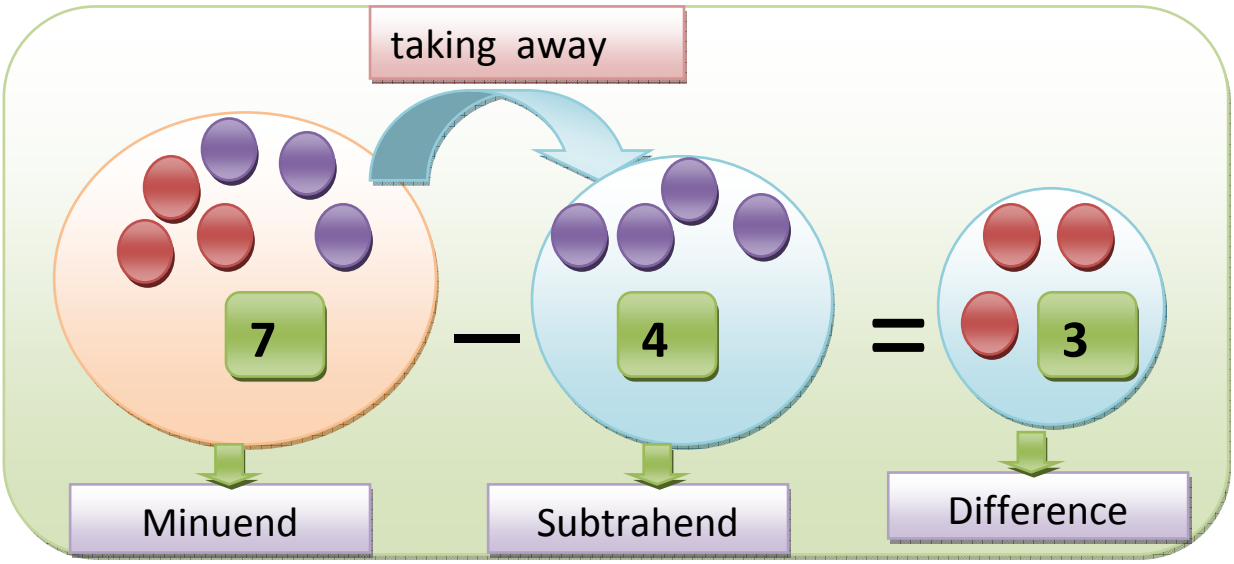
Concept building activity for double digit addition with carryover:

After the concept building of double digit addition there should lots of practice sessions. If a child face difficulty with the concept then teacher should give the concrete materials in the hand of the child and ask him/her to do the activity with concrete materials. Once this concept has established among children, then teacher will go for double digit addition with carryover concept. Let us take an example here for that:



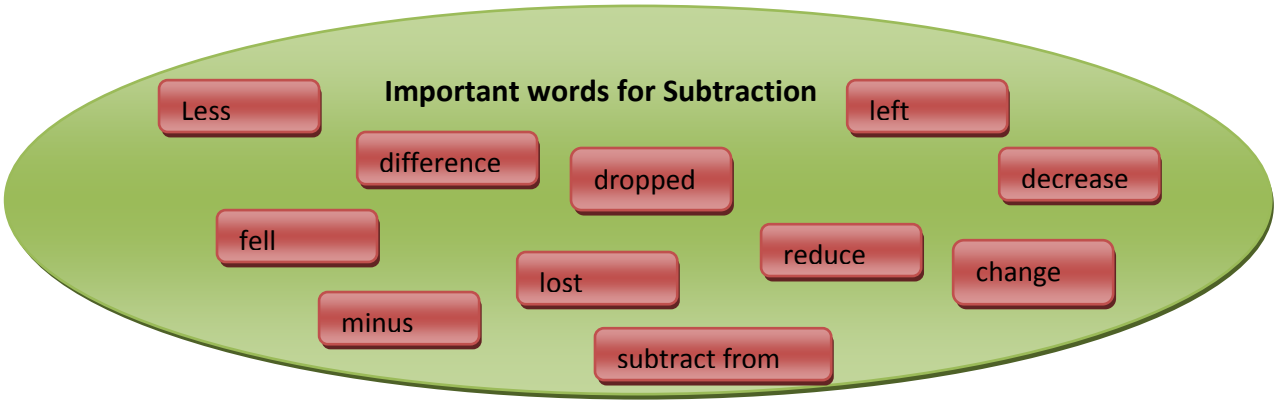
Concept of Subtraction:

Subtraction in mathematics means, taking something away from a group/set or number of things. When we subtract, what is left in the group becomes less. Notice that there are three parts to the subtraction problem shown. The part we start with is called the **minuend**. The part being taken away is called the **subtrahend**. The part that is left after subtraction is called the **difference**.



Students can use subtraction to make comparisons between what they have and what they need for a game or other activity, to budget, and to calculate remaining items as they are used, or to calculate change when a purchase is made. Teacher should encourage students to make their own word problems of subtraction. Initially teachers should help children to make these word problems in different way then gradually teacher can reduce his/her support in the process. Like other operations language plays an important role in case of subtraction and understanding the concept of the same.

Unlike addition, the subtraction has not **commutative properties.** As example, 8 – 5 = 3, 5 – 8 do not equal the same value. In other words if we change the order of numbers in the subtraction we will not get the same result.

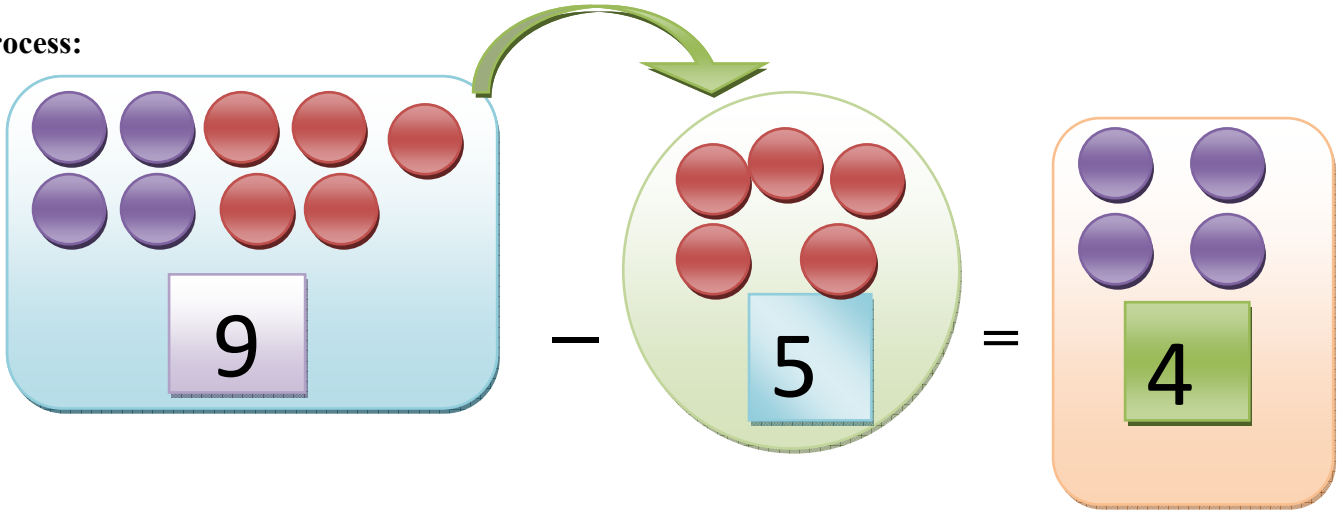


Activities for Subtraction:

Concept building activity:

The teacher will ask a child as volunteer, she will give him/her some counters and then ask to count. Suppose the child count the counter and said 9, then teacher will ask to give 5 counters to his/her friend. The child will count five and give this to his/her friend. Now teacher will ask further to count the remaining counters in his/her hand. Stone, match sticks, Channa etc which are available in the context can be used as counters.

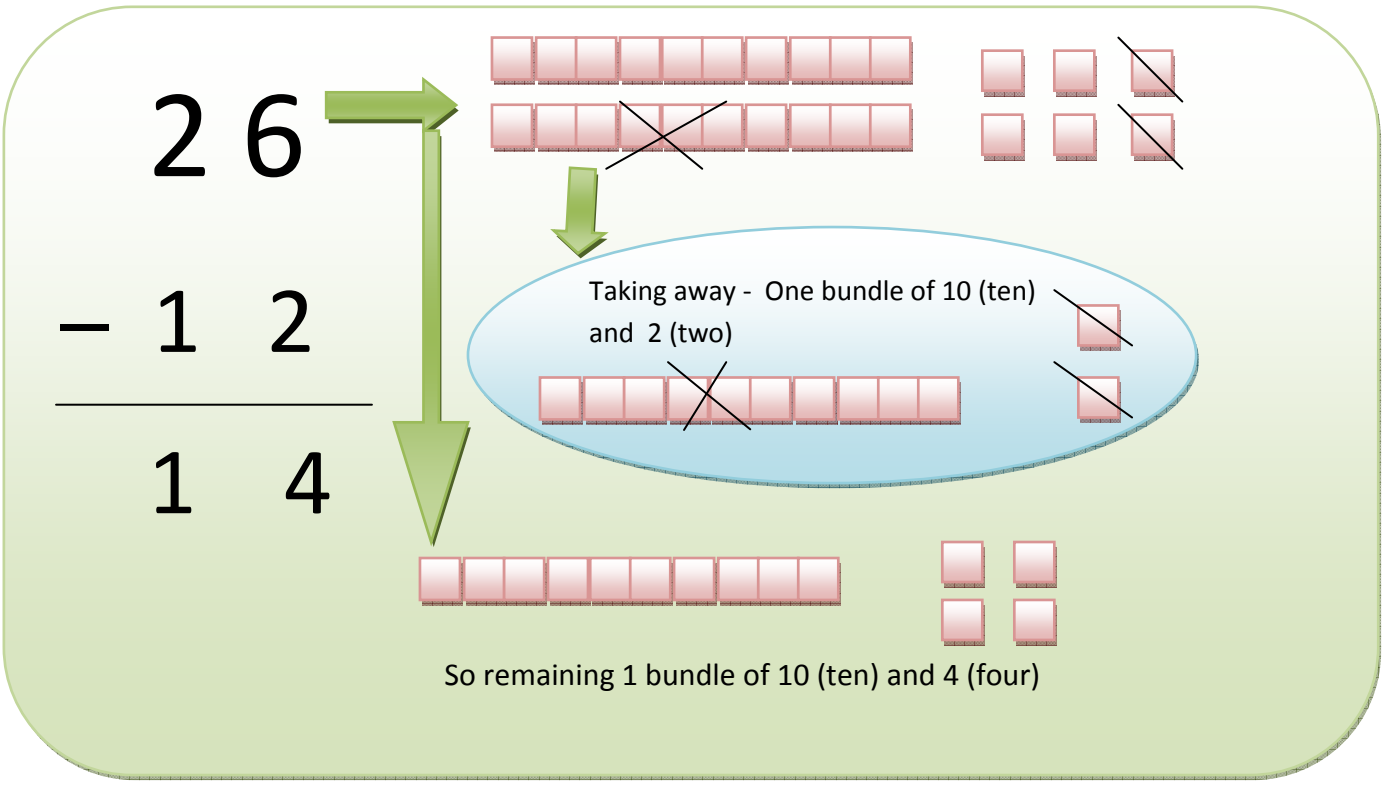
Process:



Double digit subtraction (without exchange)

The teacher will introduce the concept with the bundles of 10. Let us take an example –

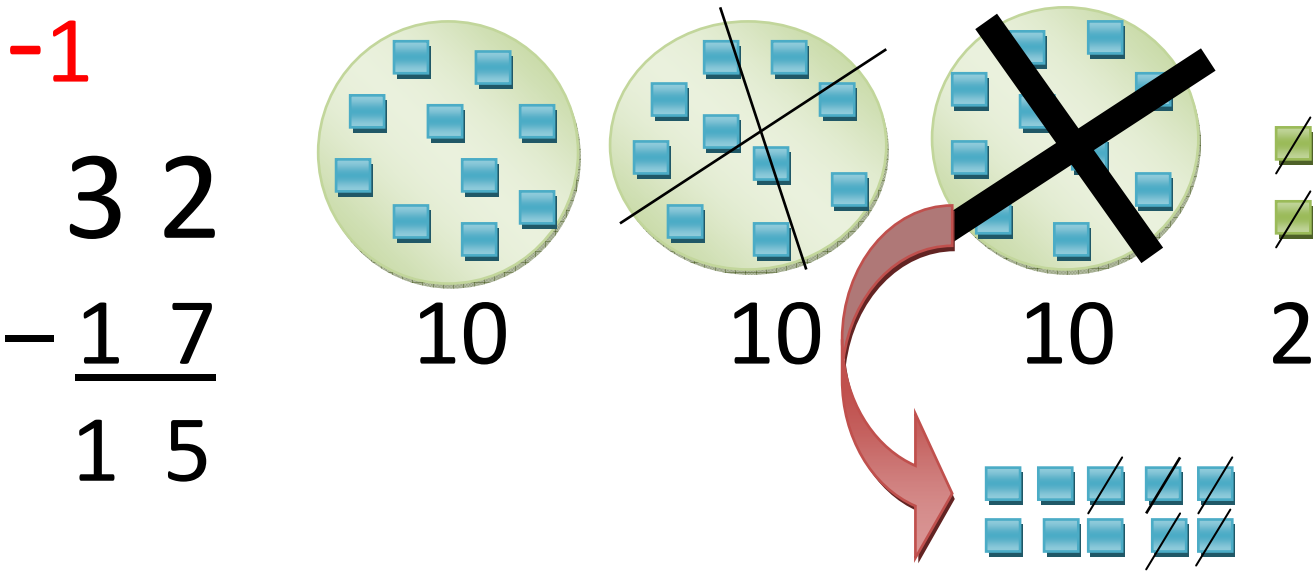
26 – 12 =?



Double digit subtraction (with exchange)

The concept of exchange is very important this can be done with the help of currency also in the similar way. We should use lots of language inputs while developing this concept. Best way to introduce this as a word problem. Like you have three packets of chocolates in each packet there are 10 chocolates and you have two loose chocolates also. Now first 7 of your friends came and you wanted to give them 1 chocolate each, and you have to give one full packet of 10 chocolates to your sister who will distribute this to her friends. Now tell, after giving 17 chocolates this way, how many chocolates are there with you?

Teacher can draw this problem in the board or can use concrete materials to make children understand the concept. This example will make them understand the subtraction with exchange concept. Example: **32 — 17 =?**



We have 3 bundles of 10 and 2 ones. Now we should give 7 ones, which we don't have. Therefore we have to break one bundle of 10. Now we shall give 7 from 12 ones. So after taking away 7 from 12 ones there are 5 ones left. We have already broken one bundle of 10, one more bundle of 10 we have to take away from the bundles of 10. So, one (1) bundle of 10 is left.

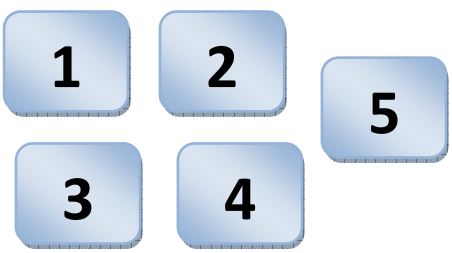
Activity for both addition and subtraction together:

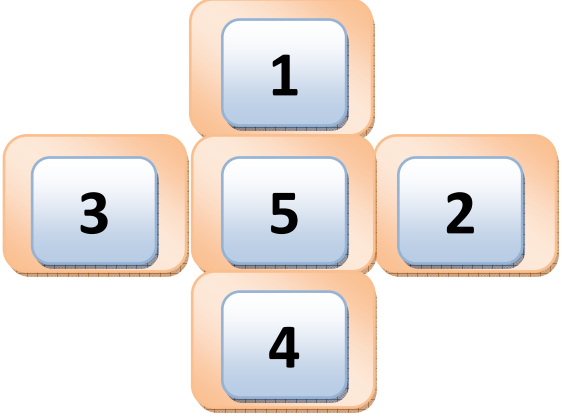
a) Dice game

This activity can be conducted between or amongst groups/ individuals. The children will take a simple dice and play the game. This is basically for mental addition and subtraction. Rule of the game: initially each group will get 10 points for starting score of the game. The group/individual throws the dice in turn. If they got 1, 3 and 5 always these numbers will be subtracted from the total score. If dice shows 2, 4 and 6 these numbers will be added to the total score. In this way the group reach 50 first they will win the game.

b) Make the equal number

This game is for addition only. The teacher will prepare number card from 1 - 5. There will be a board. The teacher will say a total and children will arrange the number accordingly. Here we are giving an example:





In the example 5 in the middle and both side, vertically and horizontally the total sum is 9. By showing this example teacher will give some task in the next.

I. For the beginners teacher will say put 1 in the middle and make total of 8 in both sides
II. Put 3 in the middle and sum will be 7 in the both side.
III. In the next ask children to explore if they put 2 or 4 in the middle do they get equal sum in the both side. From this they will find a pattern.
IV. Similarly teacher can take any 5 numbers serially and do this activity. This time teacher will not give the clue (which number should be put in the middle)

Apart from addition, this activity will help students to develop their logical reasoning and problem solving ability and discover the number patterns.

c) Travel in a bus

The teacher can give word problems which can be solved by adding and subtracting simultaneously. In a bus there were 12 passengers in the first stoppage 3 person get down and 5 persons get in. In the next stoppage 7 persons get down 3 person get in. How many passengers are there in the bus? Initially teacher help students to solve the problem step by step then gradually she/he allow students to solve these of their own.

Some common mistake in addition and subtraction:

1. **This is very common mistake in the primary level –**

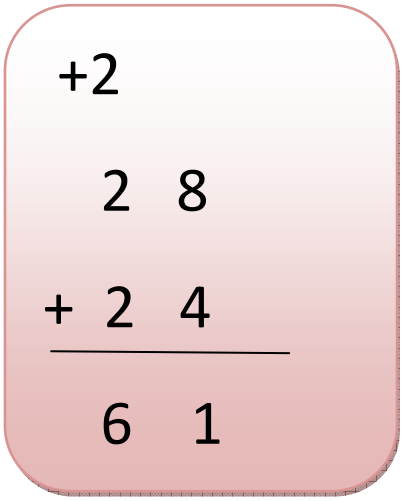
This mistake clearly said that the child can handle single digit addition. But she/he has the problem of grouping and carryover the number in the higher unit. In this case teacher should work on grouping concept (place value) and make the child understand the bundle concept of 10. Concrete materials might help the child in a better way. This is not simply a mistake of addition.

$$\begin{array}{r} 28 \\ +14 \\ \hline 312 \end{array}$$

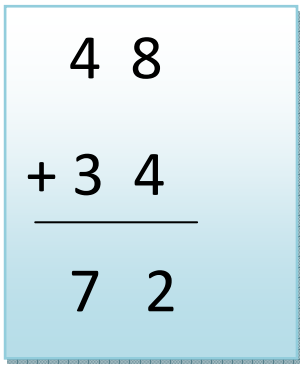
2. This problem stated in the below is purely a problem of place value not the problem of addition. In this case a teacher should work on place value concept and help children to arrange the numbers properly as per their ‘place’.

$$\begin{array}{r} 121 \\ +34 \\ \hline 461 \end{array}$$

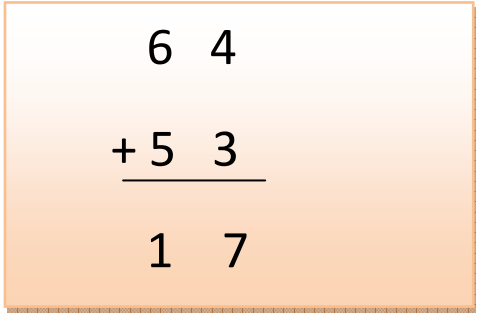
3. The students can carry the number but without understanding the concept of grouping 10. See this example below, the student arrange the numbers as per place value but when she added unit column she got 12 and she carried 2 instead of 1 ten. So this arrangement might mechanical. The teacher can ask the child to add 8 + 4, with concrete objects (beads/stone/ straw). The result will be 12 now she can as the student to exchange the 10 beads with one bundle of 10. In the next step she told the students that one bundle of 'ten' will be carried to the ten column.



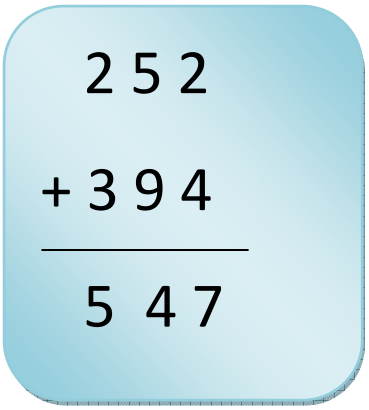
4. Students may get the regrouping process correct but forget to add what they have regrouped. In this case teacher has to analyse why this happen? Is this because of lack of attention or any conceptual gap? Accordingly she has to adopt strategy. For lack of attention practice might help but conceptual gap again the bundle concept should be introduced.



5. As students move into working with 3-digit numbers, they may not understand that the hundreds column exists when the numbers in original calculation do not contain any hundreds. Example :



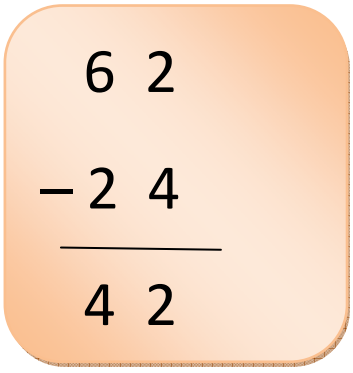
6. Students do addition and carryover from left hand side. Example –



Help children to solve the problem by using expanding notion. 252 = 2 hundred + 5 tens and + 2 ones. Similarly for 394 = 3 hundred + 9 tens + 4 ones.

Subtraction

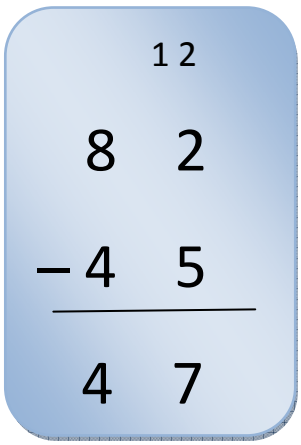
1. For subtraction this can be arise students hear that "for subtraction you always start with bigger number and subtract the smaller number." Example :-



This shows that the child is not seeing the calculation as a whole, but treating each column as a separate subtraction in its own right. The teachers can make the students understand that number 62 can be broken in different ways e.g. 50 + 12 or 60 + 2 and many more. From this concept the teacher can introduce the concept exchange.

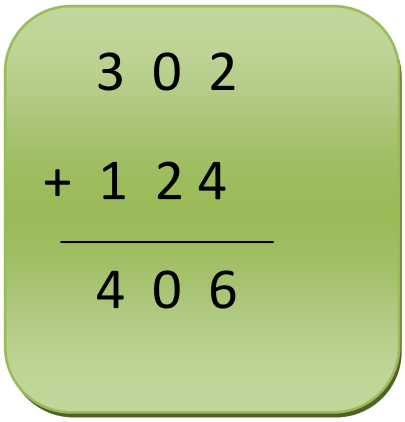
2. Pupil may try this without understanding the regrouping concept. Example :-

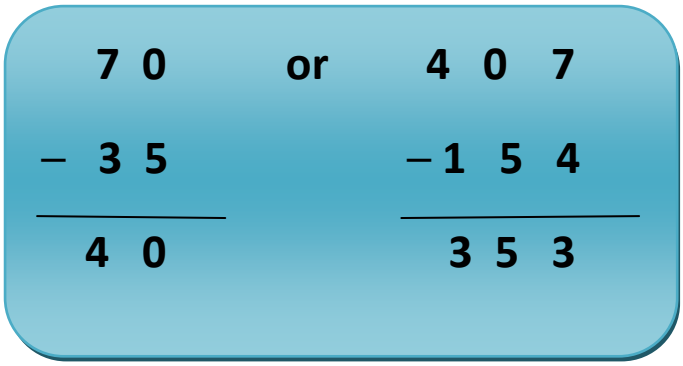
This issue can be solved by make the students understand the concept of grouping and exchange. Use of currency might help the students and make the things easier because in everyday life students are familiar with the concept exchange with currency.



Mistake because of zero as place holder:

Some common mistakes occurs when in a calculation zero is there as place holder. This is both for addition and subtraction. Let us take examples here -





In this case teacher should help students to revisit the concept of zero. In the case of carryover for addition or exchange form higher unit for subtraction she can use the expanding notion.