

Multiplication and Division

Concept:

Initially introduction of multiplication can be introduced as a problem statement to the children. Like you have 3 boxes of Lollipops in each box there are 6 Lollipops. How many Lollipops are there with you? This problem can be shown pictorially or through concrete materials.



$$6 + 6 + 6 = 18$$

Since students have previous knowledge and concept of addition, therefore they will try to solve the problem through addition. From their solution the teacher will say, "See here we are adding 6 (same number) 3 times". So we may express this as –

3 times 6, means $3 \times 6 = 18$, 'x' This is symbol of multiplication

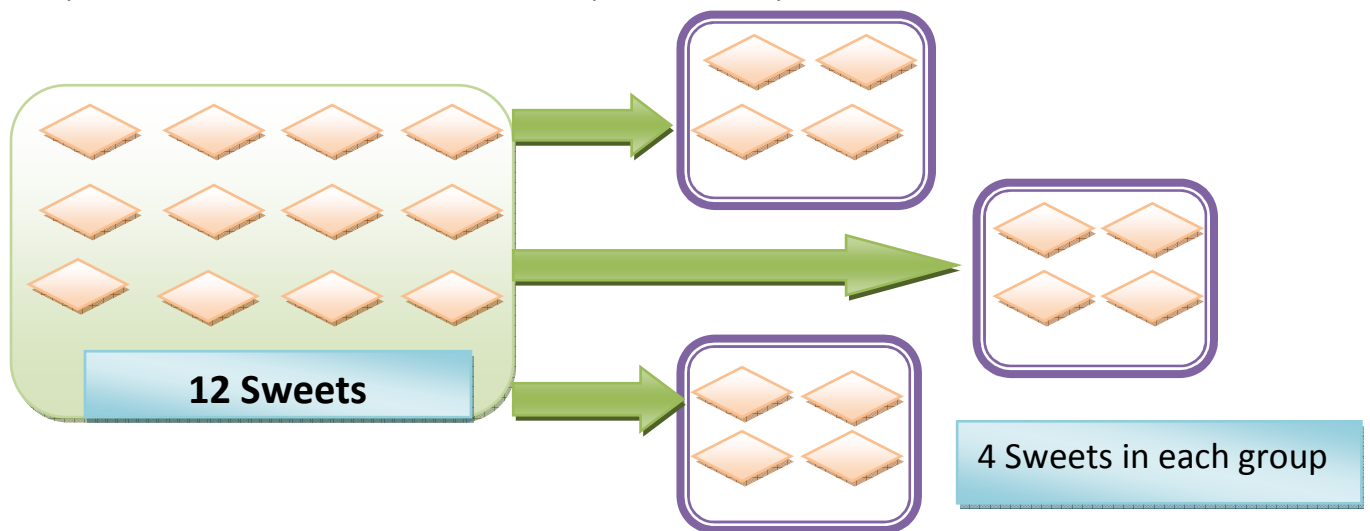
When we add **same number** more than one time then we may express that as multiplication. '**Same number**' concept need to be emphasised so that students may understand that only addition with same number can be expressed as multiplication. So multiplication is not simply the short form of addition.

Similarly in the case of division the child should understand when '**same number**' subtracted from a particular number for more than one time and then this subtraction can be expressed as division. Let us take an example – Reshmi had 12 laddus with her and there were two friends of Reshmi. Now Reshmi decided that all the Laddus should be divided among them. So there are 3 person and 12 laddus need to be divided equally. This problem can be done with the help of concrete materials and then pictorially.

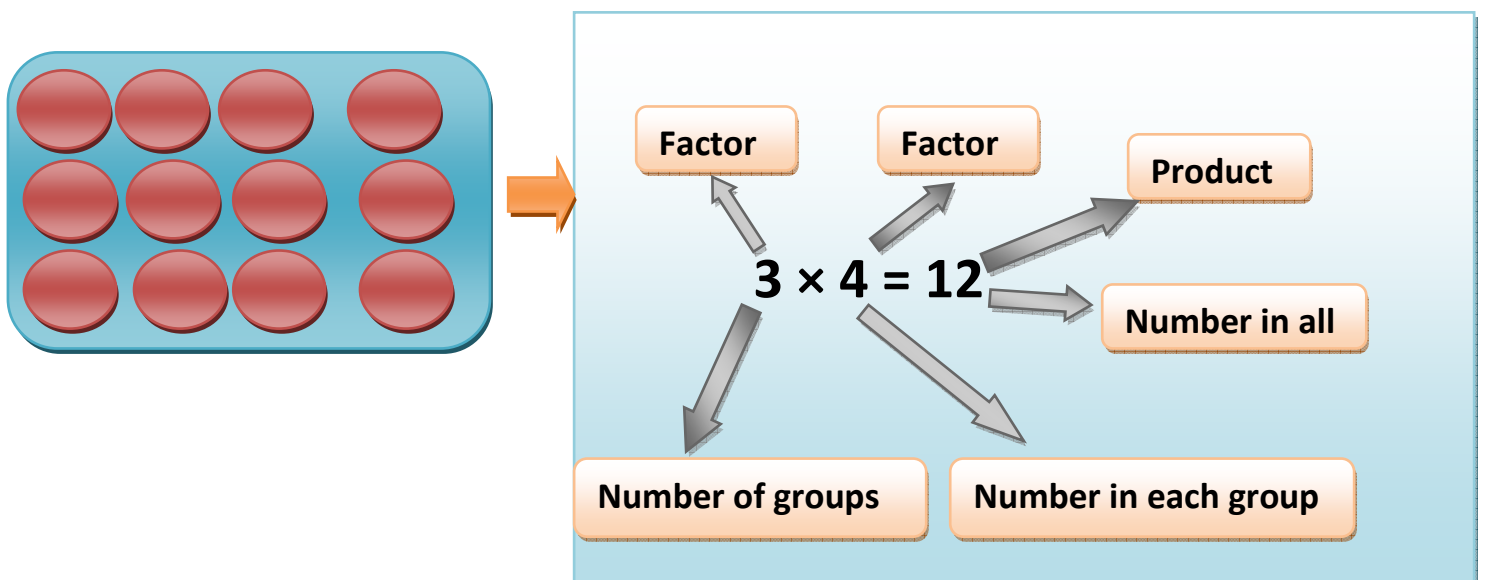
This can be shown as division here same number 3 has been subtracted from 12 for 4 times. Again students have previous knowledge and understanding about subtraction taking that as a reference point we may introduce the division. Here, $12 \div 3 = 4$, ' \div ' this is the sign of division.

The students will learn division in two ways, partitioning and measurement. When we divide to find out number of objects in each group, the division is called fair sharing or partitioning. In other hand when we divide to find out number of groups, the division is called measuring or repeated subtraction (with same number). The above example is for measuring or repeated subtraction. Now let us understand the partitioning or fair sharing division by taking an example.

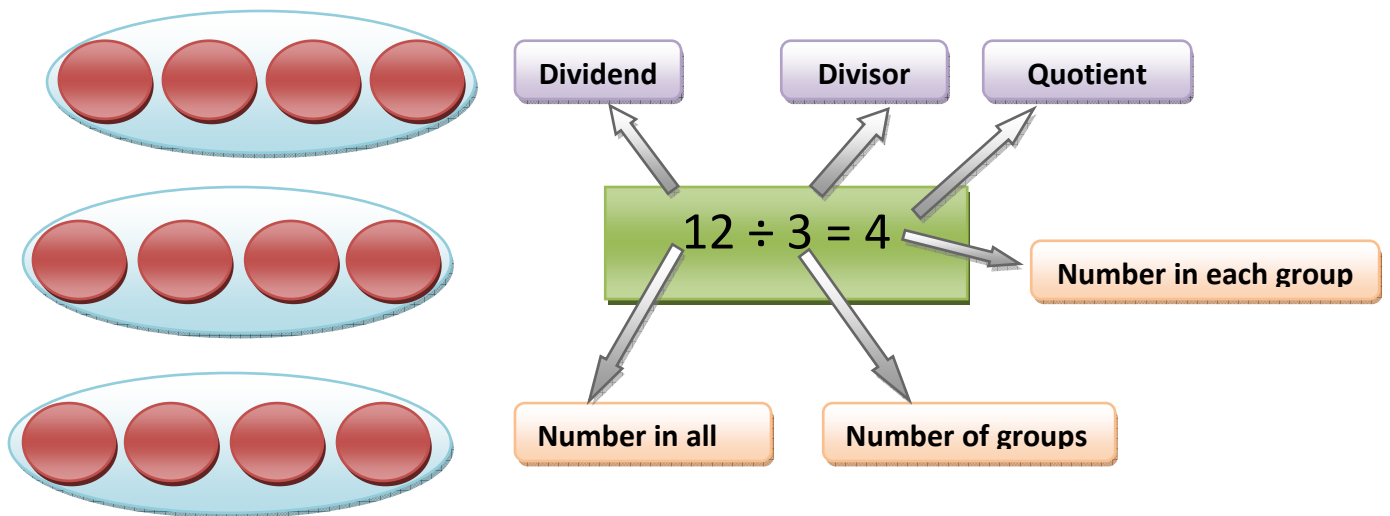
Samina has 12 sweets and 3 boxes, now she has to put equal number of sweets in each box. How many sweets will be in each box? Let us see the problem visually and solve it.



Division is the inverse or opposite operation of Multiplication. We can use **array** to make our students understand how multiplication and division are related. In multiplication we find the product of two factors, in division we find the missing factor if other factor and product are known. Manipulatives and visual aids are important while building the concept of multiplication and division. Array can illustrate both multiplication and division and thus help children to understand the concepts and relation between these. Let us take an example -



Division



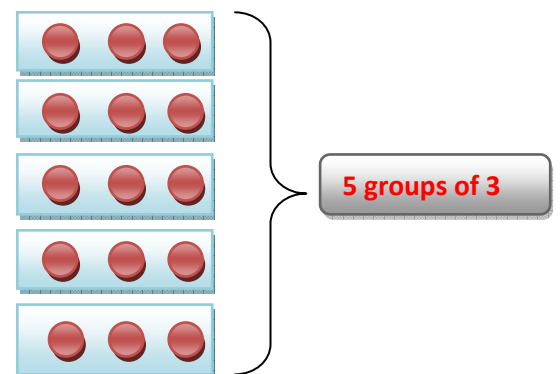
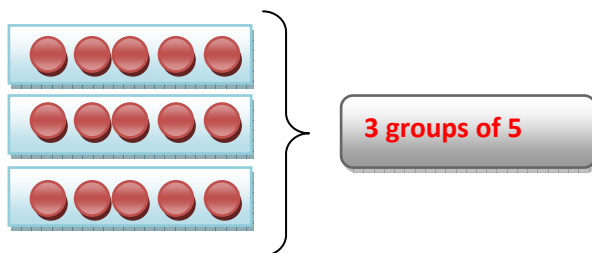
Some popular model for multiplication:

Apart from the most commonly used algorithm of multiplication, one should use some models at the foundational stage to make the students understand the concept of multiplication with some visual representation. We already talked about **array** model which helps student to understand grouping very clearly through visual representation. Let us take an example –

a) $3 \times 5 = 15$

b) $5 \times 3 = 15$

In both the cases the product is same (15) but in the first case this is 3 times 5 and in the 2nd case 5 times 3. You have 3 boxes where in each box there are 5 mangos and you have 5 boxes where in each box there are 3 mangos. These have very different practical impact in our life. A student will not understand this unless we draw some visual image for them.

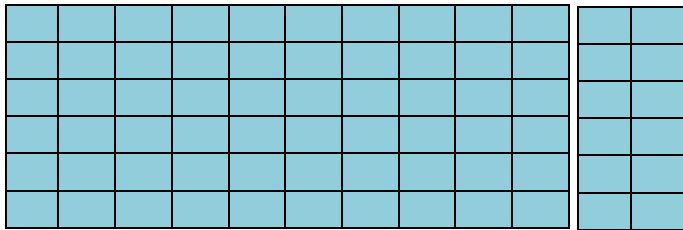


Now this is very clear that although the product are same here but the factors represent very different notion. And there is a practical implication of this. Array model is one of the popular models for multiplication and division.

Area Model:

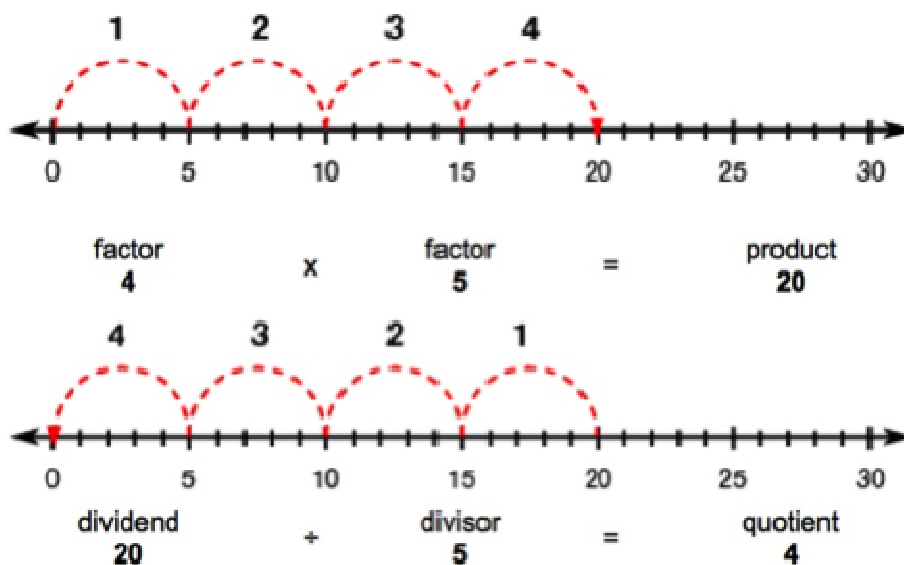
The area model is closely linked to the computation used when computing with the standard algorithm. The difference is the visual representation and connection to the Base 10 system and place value. Students can visually understand the actual size of each computation and learn how to interpret the partial products. Let us take an example:

$6 \times 12 = ?$ This can be written as
 $6 \times (10 + 2) = ?$



$6 \times 10 = 60$ **+** **$6 \times 2 = 12$** **= 72**

Number line: We may use number line and help our student to understand the concept of multiplication through visual representation.



This number line concept can be used in 'Ganeet Mala' also. The teacher can easily show 3 times 5 or 5 times 3 in 'Ganeet Mala'. Similarly if we want to divide 20 in group of 5 in 'Ganeet Mala' then we will get 4 groups. The teacher can prepare this number line or Ganeet mala at local level.

Properties of Multiplication:

- a) **Commutative property of Multiplication:** Changing the order of factors doesn't change the product. For example: $4 \times 5 = 5 \times 4 = 20$
- b) **Associative property of Multiplication:** Changing the grouping of factors doesn't change the product. For example: $(4 \times 5) \times 2 = 4 \times (5 \times 2) = 40$
- c) **Identity property of Multiplication:** The product of 1 and any number is that number. For example: $8 \times 1 = 8$
- d) **Distributive property of multiplication:** The distributive property of multiplication states that when a number is multiplying by sum of two or more numbers, the first number can be distributed to all these numbers and multiply separately which gives the same product. For an example: $3 \times (4 + 5) = (3 \times 4) + (3 \times 5) = 27$

Properties of Division:

- a) **Division by 1 property:** If we divide any number by 1 the quotient is the number itself. Example: $345 \div 1 = 345$
- b) **Division by itself property:** If we divide a number by the number itself, we always get 1 as quotient. Example: $345 \div 345 = 1$
- c) **Division any number by 0 property:** Division of any number by 0 is meaningless. Example: $345 \div 0 = \text{meaningless / undefined}$
- d) **Division of 0 by any number property:** 0 divided by any numbers gives always 0 as the quotient. Example: $0 \div 234 = 0$

Key words for Multiplication

Multiplied

Times

Area

Volume

Total

Double

Tripled

Product

Key words for Division

Equal share

Divide evenly

Quotient

How many times

Split

Part

Ratio

Average

Multiplication with two digit number:

24 → 1 time

24 → 2 times

24 → 3 times

24 → 3 times = 72 or $24 \times 3 = 72$

This above table is showing that, how visually we can make understand students the multiplication of 24×3 . Initially we may use some concrete materials and work on this concept. Then we may go with symbol but always do this with place value concept to avoid mistake in the grouping.

$24 \times 3 = ?$

2 4 x 3

$4 \times 3 = 12$, 4 ones x 3 = 12 ones. means 1 ten and 2 ones

$20 \times 3 = 60$, two ten x 3 = six

Total, 1 ten and 2 ones + 6 tens = 7 tens and 2 ones = 72

In case of multiplication of single digit with three digit number we should encourage student to do this with the place value concept initially. Let us take an example: $123 \times 6 = ?$

1 2 3 x 6

$3 \times 6 = 18$

$20 \times 6 = 120$

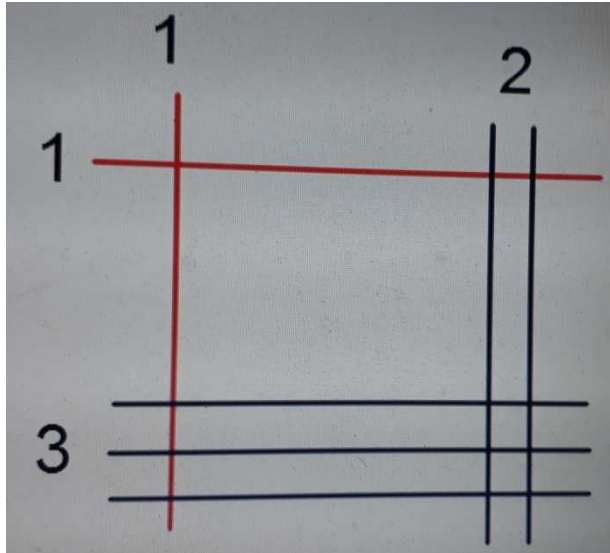
$100 \times 6 = 600$

$600 + 120 + 18 = 738$

600
120
+ 18
<hr/>
738

Multiplication by using straws/ sticks:

This activity is quite helpful for multiplication and its table. Take straws of two colour or you may take some sticks of any two colour, which is available in your context. Now multiplication is easy for your students. Let us take an example: $12 \times 13 = ?$ See in the picture red straw representing

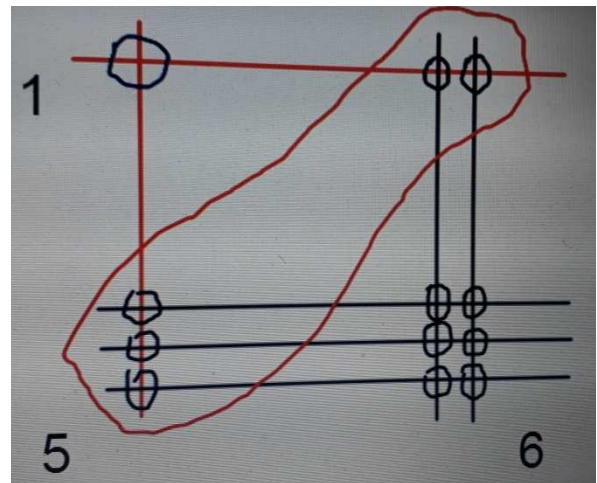


bundle of 10 black straws representing unit (ones). Here for 12 we took one red straw 10 and 2 black straws, similarly for 13 one red straw (10) and 3 black straws. These straws should be arranged like this and then just count the junction these straws made. Here only black straws made six junctions which tell us the unit number is 6. Black straws and red straws made 5 (3 + 2) junctions all together. This indicates the ten column number is 5 and finally red straws made 1 junction which indicates the hundred column. So, $12 \times 13 = 156$.

This method is very popular in China and some other Asian countries. This algorithm can be explained as –

$$\begin{array}{r}
 12 \\
 \times 13 \\
 \hline
 156
 \end{array}$$

or 1 5 6

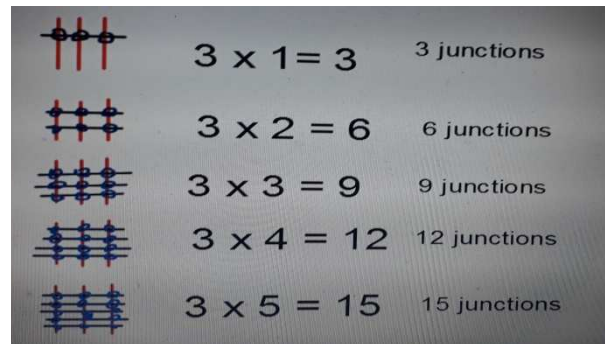


Multiplication Table:

Fluency in multiplication table is extremely necessary for further mathematical operations and daily life. In our traditional classroom, most of the cases children are asked to memorise the multiplication table. Many of our school memories associated with chorus chanting of multiplication table standing in the school courtyard. We should avoid this rote learning and encourage the children to construct and reconstruct multiplication table of their own. If students can add single-digit number to a two-digit number, they can at least reconstruct their tables even they have not yet developed fluency. Therefore we should ensure that students can add fluently before begin to learn their tables. We should also encourage them to use some concrete materials and construct their tables.

Multiplication table by using straws/ sticks

Teacher can use two sets of sticks/ straws with two different colour and help students to construct their own tables. We are taking the example of table of 3 here. The students will simply arrange the straws or sticks as the arrangement has shown here and count the junction formed with the arrangements. Similarly the students do tables with all the numbers. Once the students are confident with this work then the teacher can

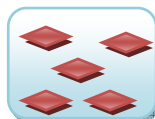
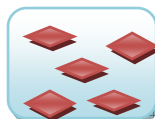
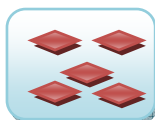


गुणा के लिए चार्ट		1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	
2	2	4	6	8	10	12	14	16	18	20	
3	3	6	9	12	15	18	21	24	27	30	
4	4	8	12	16	20	24	28	32	36	40	
5	5	10	15	20	25	30	35	40	45	50	
6	6	12	18	24	30	36	42	48	54	60	
7	7	14	21	28	35	42	49	56	63	70	
8	8	16	24	32	40	48	56	64	72	80	
9	9	18	27	36	45	54	63	72	81	90	
10	10	20	30	40	50	60	70	80	90	100	

introduce the multiplication table. Here we are giving a sample of multiplication table (10 x 10) chart. The teachers can use the simple square copy available in the market for mathematics practice. The main objective here is give opportunity to students to make their own multiplication table. If in any point of time they face problem then teacher will give concrete objects e.g. straws, stick etc and ask them to do the table with concrete materials and then write in the chart. This activity will help students to discover number patterns which will further develop their better computational skills. These number patterns also build the concept of divisibility of numbers, LCM etc in the future. This activity can be done in the small group or at individual level.

Multiplication with Zero:

Any number multiplied by Zero will give the product 'zero' always. Best way to make this concept understand to the children, one should use concrete materials and use language problems. As example – There is **3 boxes** in each boxes there are **5 sweets**. So there are **3 x 5 = 15 sweets**. There are 3 boxes but no sweets are there in the boxes. How many sweets are there altogether? This means **3 boxes x 0 (no sweets) = No sweets (0), 3 x 0 = 0**.



3 boxes x 5 sweets = 15 sweets, 3 x 5 = 15



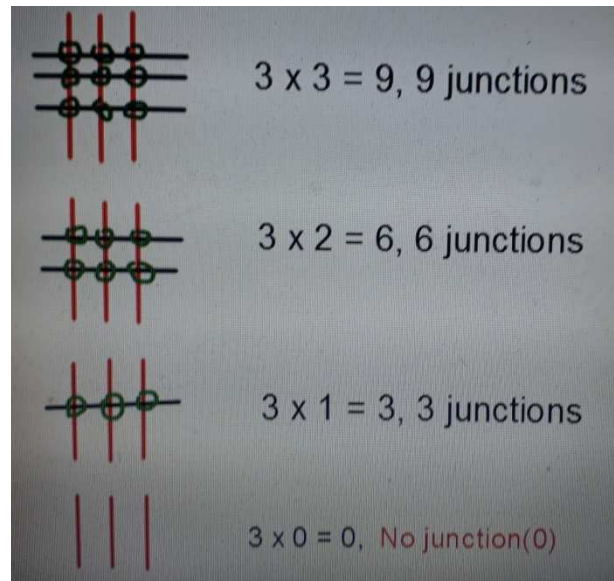
3 boxes x 0 sweets = 0 sweets, 3 x 0 = 0

The multiplication with zero is a very important concept for young children. A teacher should design different activities to make this concept understand to the children. The teacher can use sticks or straws and build this concept among children. This is suggested by many mathematics educators that the entire table should start from multiplication with **zero**. As example –

$2 \times 0 = 0, \quad 3 \times 0 = 0$

$2 \times 1 = 2, \quad 3 \times 1 = 3$

$2 \times 2 = 4, \quad 3 \times 2 = 6$

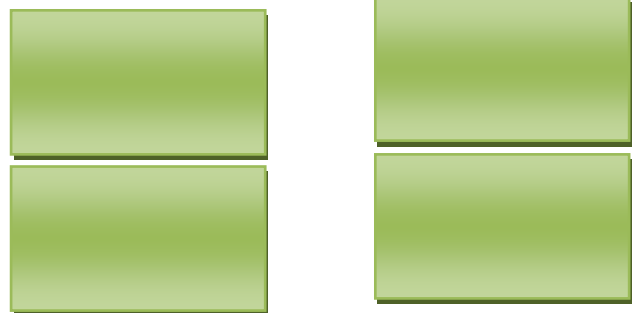
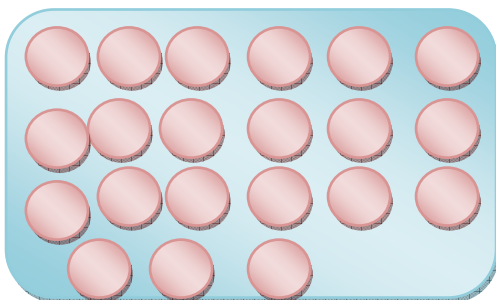


This will help students in the process of division with zero.

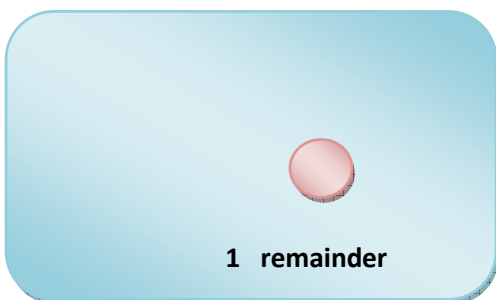
Concept of division with remainder

This concept should be introduced with problem statement and then give opportunity to solve the problem with concrete materials/objects. You have 21 laddus 4 boxes are there. Now you have to distribute laddus equally in 4 boxes. How many laddu will be in each box and how many left? Now see this problem pictorially –

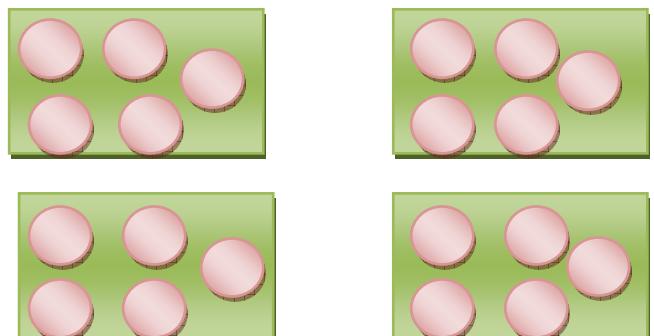
1. Here is the problem; you have to **divide 21 laddus in 4 boxes** equally.



2. The solution could be -



In each box there are 5 laddus



We already introduce the game 'Mama gaye Bazar' the concept of division can be explained through this game quite nicely. Ask 21 children to participate in the game. They will move into a circle and tell 'mama gaye bazar' teacher will ask 'keya laye bhai keya laye'. The children will respond 'mama laye laddu' and the teacher will further ask 'kitne bhai kitne?' Students will respond 'aap chahe jitne'. Then teacher will call 5 (panch panch). The children will make group of five and get 4 groups of 5 and one child will be left. So in this way a teacher can introduce any division with remainder and students will learn this through this fun game.

Standard Algorithm of division

1 1 4 one hundred one ten and four

$$\begin{array}{r}
 4 \overline{) 456} \longrightarrow 400 \div 4 = 100 \\
 \underline{-4} \\
 0 \longrightarrow 50 \div 4 = 10 \\
 \underline{-4} \\
 1 \longrightarrow 16 \div 4 = 4 \\
 \underline{-1} \\
 0
 \end{array}$$

In division estimation is very important. When a child dividing 456 (four hundred fifty six) by 4 then she should estimate that 'quotient' would be one hundred and above.

Concept of division with '0' and '1'

Division with zero (0) and one (1) is always challenging concept to make the students understand. When students understand the concept of division, they can proceed to explore the rules for dividing with 0 and 1. It is better to encourage the students to explore and discover the rules by their own. The teacher should use some counter and any concrete objects which are available in the local context. Let us take some examples:

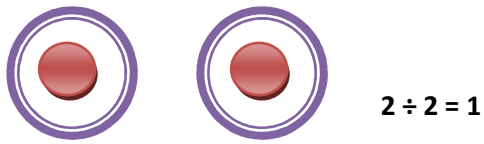
Divide 4 counters into 4 groups



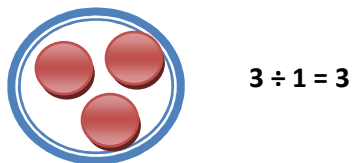
$$4 \div 4 = 1$$

A teacher should make and tell some story to the children to explain the concept. As example - Ravi's father brought 4 sweets and asks Ravi to divide these among 4 family members. Ravi gave 1 each to each member.

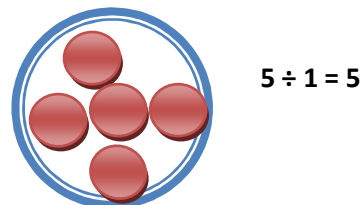
Divided 2 counters into 2 groups



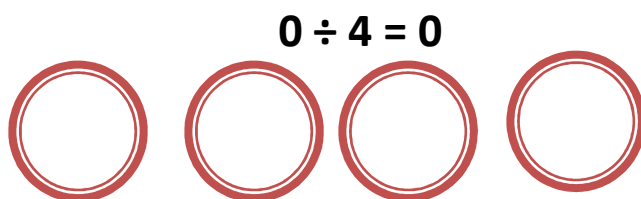
Put 3 counter in 1 group



Put 5 counters in 1 group



The concept of division with zero (0) should be introduced with a story. In a village there was a rich man named Harihar Pandit. He had 4 sons and one day he called his sons and said, I have some money in my bank account and after my death you take all the money from bank and divide equally. After his death all 4 sons visited to bank and ask bank manager for the money. Bank manager checked their father's account and said, "Your father donated all the money to a trust before his death, so there are 0 balances in the account". So Rs. 0 will be divided among 4 sons.



0 counters divided into 4 groups. The teacher can ask children if you don't have any counters in your hand and I asked for divide these into 4 groups then there will not be any counter in any group. Similarly if we divide 0 by any number the quotient will be 0 always.

Standard division algorithm with zero

$$\begin{array}{r}
 101 \\
 3 \overline{) 303} \\
 \underline{-3} \\
 00 \\
 \underline{-0} \\
 03 \\
 \underline{-3} \\
 0
 \end{array}$$

A teacher should encourage students to do this step by step. He/she will say student when zero (0) come from the result of subtraction that zero will not divide but if you take zero (0) from the dividend then that zero (0) will be divided. Once again estimation is very important aspect to minimize to mistake in this type of division.

Standard algorithm of division with remainder

$$\begin{array}{r}
 085 \\
 \hline
 5 \overline{) 426} \\
 \underline{-0} \\
 42 \\
 \underline{-40} \\
 26 \\
 \underline{-25} \\
 1 \text{ (remainder)}
 \end{array}$$

In traditional classroom we said since 4 is smaller than 5 then we have to take up to two digits here. But this is 4 hundred this is not smaller than 5. If we ask or pose a question to the children, how many times 5 are there in 4? The possible answer would be not even 1 time, 0 times. So in hundred columns this is 0. Then as per rule of this algorithm we have to subtract the number and take next number from the dividend. Here we subtracted 0 from 4 and then took 2 from the dividend. Now how many times 5 are there in 42? The answer would be 8, then after subtraction we will take next digit 6 and further ask, how many time 5 are there in 26? The obvious answer would be 5 and 1 remainder. So after doing the division with concrete materials, a teacher should help her student by clarifying each steps in standard algorithm of division.

Common errors/ mistakes made by students in multiplication and division:

i)

$$\begin{array}{r}
 23 \\
 \times 7 \\
 \hline
 1421
 \end{array}$$

Problem: Carry forward is not done

Analysis: Place value concept is not clear

How to help the student?

Need to work on place value concept. In 23, 3 is unit (ones) and 2 is in ten place mean 20. So when we are multiplying $3 \times 7 = 21$ we are getting 2 tens and 1 and when we are multiplying $20 \times 7 = 140$ means 2 ten x 7 times = 14 tens, So all together 16 tens and 1 or $140 + 21 = 161$

ii)

$$\begin{array}{r}
 335 \\
 \times 5 \\
 \hline
 1575
 \end{array}$$

Problem: Forget to add carryover at hundred place

Analysis: Careless mistake/ lack of attention or carry over concept have not formed properly, copied from friend's copy.

How to help the student?

Give similar task for confirmation the problem. For lack of attention more and more practice and carry over concept expand and multiply – 5 ones x 5 = 25 ones means 2 ten and 5, then $30 \times 5 = 150$ means 1 hundred and 5 tens, and finally $300 \times 5 = 1500$, means 15 hundred. Then add all these above.

iii)

$$\begin{array}{r} 23 \\ \times 12 \\ \hline 46 \\ + 23 \\ \hline 69 \end{array}$$

Problem: Product is not carried over properly

Analysis: Lack of understanding this multiplication algorithm, place value concept is not clear.

How to help the student:

Use expanded form to carry out the product. $23 \times (10 + 2)$
 $(23 \times 10) + (23 \times 2) = 230 + 46 = 276$. Make the child understand 1 in 12 is not one this is 1 ten, and 2 in 23 is not two this is 2 ten. When we multiply 10×10 , we get hundred as product. So when we are saying here 2×1 , actually we are multiplying 20×10 times = 200 (two hundred).

iv)

$$\begin{array}{r} 202 \\ \times 4 \\ \hline 848 \end{array}$$

Problem: Did not handle the multiplication with zero

Analysis: Multiplication with zero is not clear and do not know what to do then writes $0 \times 4 = 4$.

How to help the student: Need to take concrete materials and build the concept of multiplication with zero. Teacher might say, I am giving you zero (0) sweets 4 times, now tell me how many sweets you have? I am giving 0 sweets mean; I am not even giving you single sweet. So, $0 \times 4 = 0$ or $0 + 0 + 0 + 0 = 0$, if you add zero 4 times you will get zero only.

v)

$$\begin{array}{r} 2 \\ 3 \overline{) 600} \\ \underline{6} \\ 00 \end{array}$$

Problem: Division is not carried out properly

Analysis: Lack of understanding in the process of division

How to help the student:

For division divide the highest unit first. Here highest unit is 6 hundred.
 So, $6 \text{ hundred} \div 3 = 2 \text{ hundred}$
 $0 \text{ tens} \div 3 = 0 \text{ tens}$
 $0 \text{ ones} \div 3 = 0 \text{ ones}$, Therefore quotient will be 200.

vi)

$$\begin{array}{r} 11 \\ 5 \overline{) 505} \\ \underline{5} \\ 05 \\ \underline{5} \\ 0 \end{array}$$

Problem: Did not able to handle '0' as place holder, therefore division is not carried out properly.

Analysis: Lack of understanding to divide $0 \div 5$

How to help the student: $505 \div 5$ means quotient will be not less than hundred.

$5 \text{ hundred} \div 5 = 1 \text{ hundred}$

$0 \text{ tens} \div 5 = 0 \text{ tens}$

$5 \text{ ones} \div 5 = 1 \text{ one}$, Therefore quotient will be 101. In case further clarification needed, then teacher will use 0 counters in 5 bowls. So with concrete materials or through stories students will understand the concept of $0 \div 5 = 0$.

vii)

$$\begin{array}{r} 12 \\ 4 \overline{) 481} \\ \underline{-4} \\ 8 \\ \underline{-8} \\ 1 \end{array}$$

Problem: Division is not carried out properly

Analysis: Lack of understanding on process of division

How to help the student: $481 \div 4$ means quotient will be not less than hundred.

$4 \text{ hundred} \div 4 = 1 \text{ hundred}$

$8 \text{ tens} \div 4 = 2 \text{ tens}$

$1 \text{ one} \div 4 = 0 \text{ ones}$, Therefore quotient will be 120 and 1 remainder will be there.

Self Assessment:

1. Tick (✓) the correct option

Division and multiplication are –

- a) similar operation
- b) inverse operation
- c) unary types operation

2. Tick (✓) the correct option

Commutative property of multiplication is –

- a) equally applicable in the division
- b) partially applicable in the division
- c) not applicable at all

3. Tick (✓) which statement is wrong here

- a) If any factor of multiplication is '0' then product will be always '0'.
- b) '0' divided by any number always gives the quotient '0'.
- c) Quotient x dividend = divisor
- d) Any number divided by '0' is undefined.