Azim Premji University

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DIVISION

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A VISUAL APPROACH

At Right Angles
A Resource for School Mathematics

INTRODUCTION

DIVISION

It is well known that division and subtraction are generally found to be more difficult to learn by children, as compared with addition and multiplication. Particularly in the case of division, children have difficulty both in identifying the situations requiring division and the complex formal procedure involved in the long division process. There is also the role played by zero as a place value holder in the quotient. Given that these are the three main difficulties, teachers need to slow down and exercise care while teaching the division concept. I have often seen teachers not making use of the place values of numbers while explaining division procedure. Unless the place value is emphasized, the logic of the placement of quotient in the right place cannot possibly be understood.

Do plenty of warm-up activities involving these two division contexts before introducing division:

a) Division as equal distribution;

b) Division as repeated subtraction.

(Note: Division as a rate or reducing scale factor is taught only at the upper primary level.)

***Keywords**: Division, repeated subtraction, equal distribution, equal groups, sharing, multiplication, dividend, quotient, remainder, divisor*

ACTIVITY ***ONE***

Distribution into equal groups and internalising division contexts

- Draw two line segments parallel to each other. Ask a certain number of children to distribute themselves equally along the two segments (*but first ensure that the number of children is even*). Each opposing pair can shake hands to verify the one to one correspondence.
- Draw 3 large circles on the ground. Ask a certain number of children (*but first ensure that the number of the children is a multiple of* 3) to distribute themselves equally between the three circles. Let them count out their number for verification.
- Draw a large square on the ground. Ask a certain number of children (*but first ensure that the number of the children is a multiple of 4*) to distribute themselves equally on the four sides of the square.

It is important for the teacher to let the children figure out how to do the distribution on their own. They may blunder at the start, but they will eventually come out with effective ways of solving the problem.

ACTIVITY ***TWO***

Distributing objects into equal groups and internalising division contexts

Materials required: Square pieces, straws and rubber bands, coloured buttons. Peg board and pegs or graph board and seeds

Let four children share the seeds or marbles amongst themselves equally.

The seeds or marbles can also be placed in paper plates or bowls.

Let another group of children work with straws and share out straws equally amongst themselves.

Let yet another group of children arrange square pieces in the required number of rows with the same number of pieces in each row.

Activities 1 and 2 should be done over several days with varied materials and in different contexts to provide children with a firm grounding in the division experience.

ACTIVITY *THREE*

Division as repeated subtraction

Materials required: Beads, straws

Give a child, say, 20 beads. Ask him to remove 2 beads at a time from the pile. Ask him to record the number of times he removes 2 beads. At the end let him record the statement as follows.

"20 seeds, 2 seeds removed 10 times."

Repeat this activity with other numbers, making sure that there will be no remainder left in the initial stage.

Often teachers introduce the symbol for division or the formal way of writing division too soon. While the child is still struggling to understand the division concept and internalise it, he or she is confronted with new symbols and complex recording procedures. It is best to give children exposure to activities involving division for at least 10 days before we introduce the symbol.

ACTIVITY *FOUR*

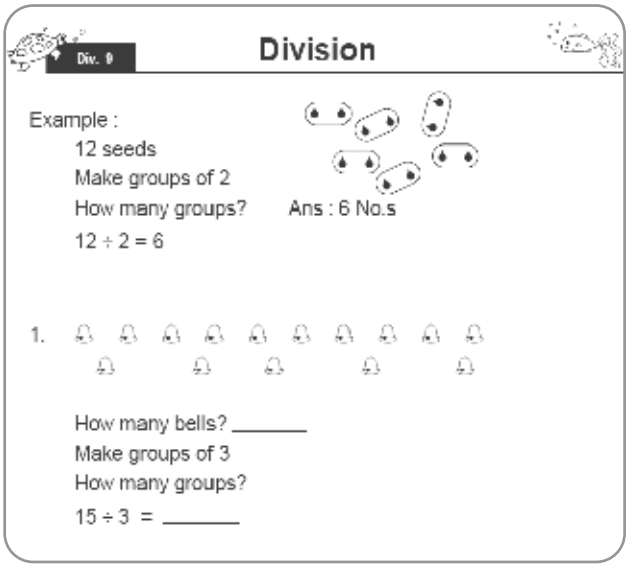
Introduction to the division symbol

Materials required: Seeds or square pieces

Let the children use pairing and count the number of pairs and record the information as shown in the picture.

Let them do repeated subtraction and count the number of times subtraction has taken place, and let them then record the information.

It is important that they record the result in both forms, as a grouping and as a division fact, till they internalize the relationship between grouping and division. The same holds if one is doing it through repeated subtraction.



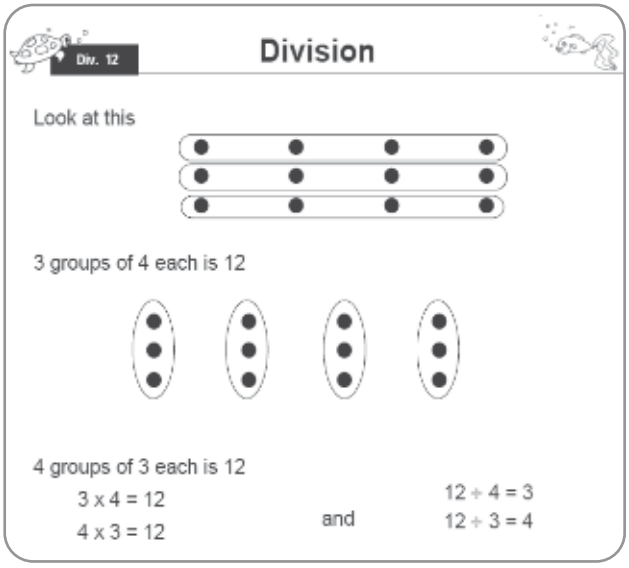
ACTIVITY *FIVE*

Helping children to see the connection between multiplication and division

Materials required: Square pieces, seeds. Peg boards with pegs

It is quite possible that children may intuitively use multiplication facts to arrive at the answers for division problems. In fact, children who have internalised multiplication concepts quite thoroughly may straight away use multiplication facts by converting the division problem into complementary multiplication problem. Example: 12 ÷ 4 may be converted to: "4 times which number equals 12?"

However, not all children may see the connection. Hence it becomes necessary for the teacher to lead the children into this discovery by asking directed questions.



How many square pieces did you have at the start? 12. Into how many rows are you going to distribute them? 4. How many pieces have you placed in each row? 3 pieces. How do we state this as a division fact?

$$12 \div 4 = 3.$$

Can you describe this arrangement (as shown in the picture) as a multiplication situation?

$$3 \times 4 = 12.$$

ACTIVITY *SIX*

To show that every division fact gives rise to another division fact

Materials required: Square pieces

Arrange square pieces in an array form as shown in the picture for activity 5.

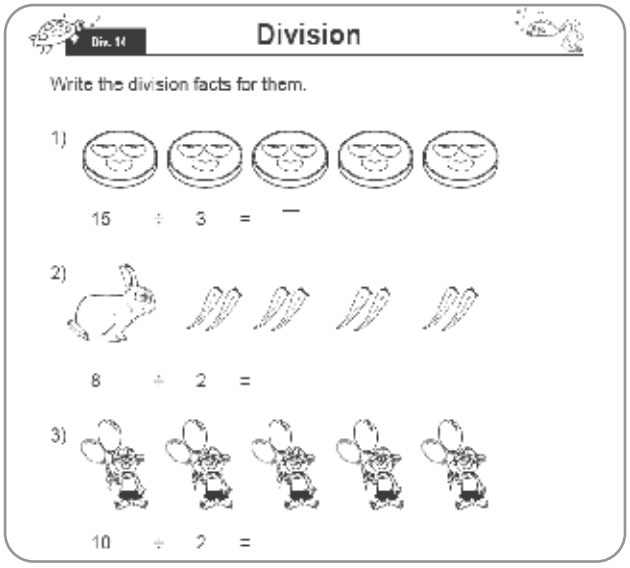
Help children to record the division fact by looking at it one way; that is, as $12 \div 4 = 3$.

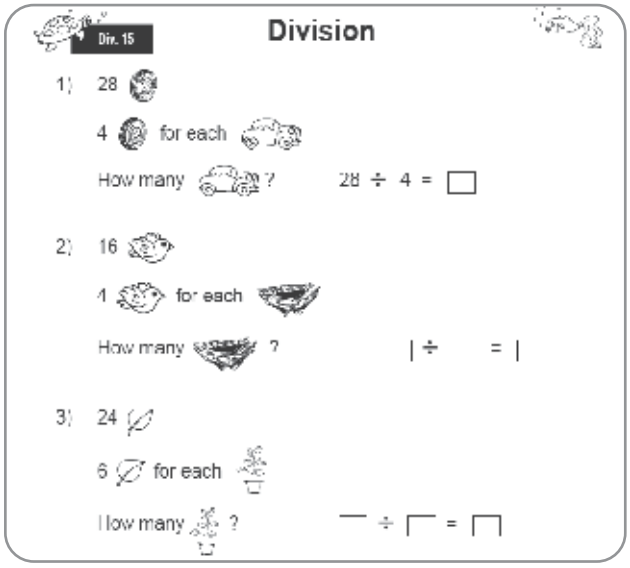
Now turn the array the other way round to record the other division fact arising from the same situation; that is, $12 \div 3 = 4$.

ACTIVITY SEVEN

To show division using plenty of visuals and creating context situations

Materials required: Square pieces





Create many visual word problems or give plenty of context situations which help children understand the division concept.

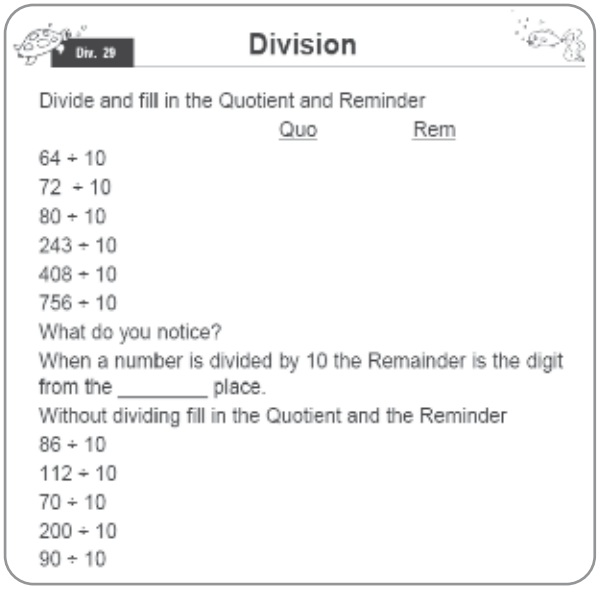
ACTIVITY EIGHT

To show the division procedure for one step division problems

Materials required: Square pieces

Take any problem, say: 16 ÷ 8. Take 16 square pieces and distribute equally to 8 people. Let children see that each person gets 2 pieces. Now simultaneously show the recording procedure, emphasizing the fact that the total number being shared (namely, 16) is written in the centre, the number of people among whom it is shared is written on the left, and the number each one gets is written on top. Use place value headings on the dividend. It is best to get children into the practice of writing the quotient on top, synchronizing it with the correct place value. This reduces the scope for errors, and zero as a place value holder becomes evident.

ACTIVITY NINE | Division by ten


Div. 29

Division

Divide and fill in the Quotient and Reminder

	Quo	Rem
64 ÷ 10		
72 ÷ 10		
80 ÷ 10		
243 ÷ 10		
408 ÷ 10		
756 ÷ 10		

What do you notice?

When a number is divided by 10 the Remainder is the digit from the _____ place.

Without dividing fill in the Quotient and the Reminder

86 ÷ 10

112 ÷ 10

70 ÷ 10

200 ÷ 10

90 ÷ 10

Show several problems with division by 10. Make a table as shown, showing the division facts along with quotient and remainder. Let children notice that in division by 10, the number in the units place becomes the remainder, and the 'rest' of the number becomes the quotient.

ACTIVITY TEN | Division of zero by a number, and division by zero

$$12 \div 0 = ?$$

This is best explained through repeated subtraction.

Ask the question: "How many times can zero be subtracted from 12 to get zero at the end"?

No matter how many times we subtract zero from 12, we will never get a zero as the answer! It goes on indefinitely. So the division cannot be done at all.

Division of zero by a number:

$$0 \div 8 = ?$$

While teaching multiplication facts through the flow technique, we had established that any number multiplied by 0 yields 0. We could now use this fact to show that the answer would be 0.

Note on terminology: It is not necessary to introduce all the words – '*dividend*', '*divisor*', '*quotient*', '*remainder*' – at the lower primary level. 'Remainder' alone will make sense, as it is a commonly used word in English. Instead, we can refer to these numbers as: How many (objects or sweets) need to be distributed? To how many people? How many will each get?

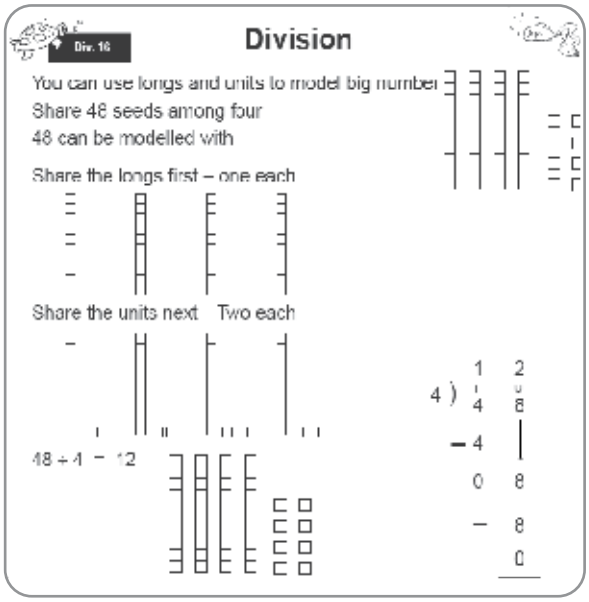
ACTIVITY
ELEVEN

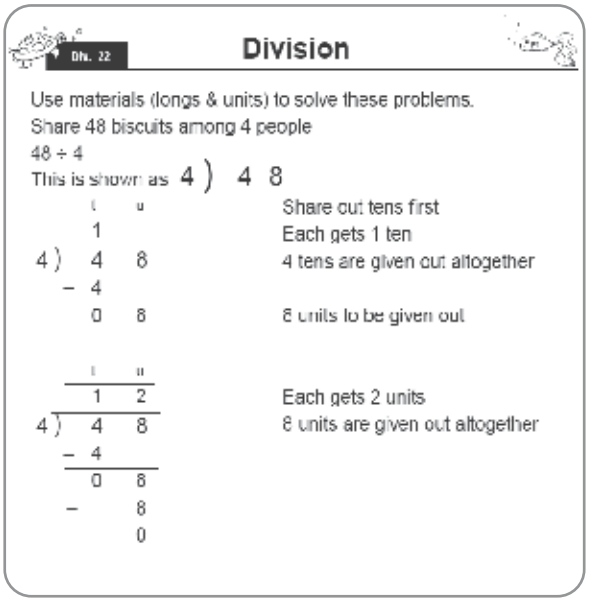
Division of a double digit number by a single digit number (without any exchange and remainder)

Materials required: Place value Kit.

Introduction to division of a double digit number by a single digit number is best done through place value material. In the first stage we introduce problems which do not require exchange from tens to units.

Example: "Share 48 rupees amongst 4 people."





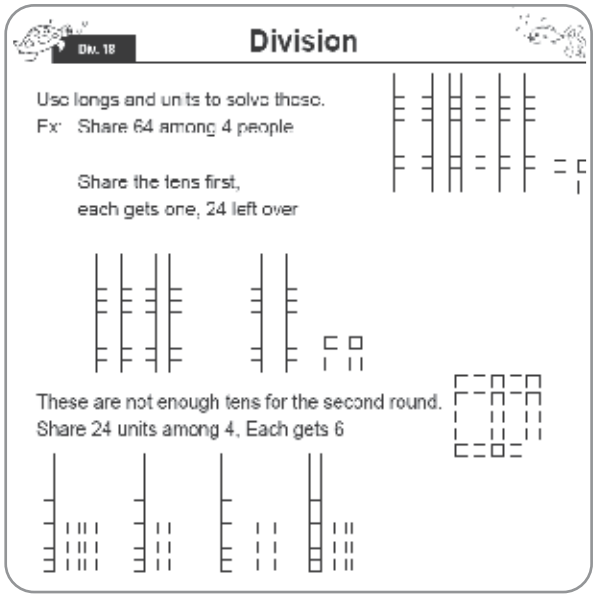
As shown in the picture, we use tens and units material to show 48. We start with **tens** (this is an important point to note as in all the other arithmetical operations we start from right to left, and it is in the division operation alone that we move from left to right) and ask the question how many tens (at each point, read the number with its place value to draw the child's attention to it) can we share out equally amongst 4 people? Each one gets 1 **ten** (*emphasize again the place value*). This is recorded in the division problem as 1 ten in the tens place over the 4. It is important to emphasise the place value all the way through. In my school days we used to write the quotient on the right hand side of the dividend. But this way one does not see the correspondence between the digits of the quotient and the dividend. Placement of zeroes as place holders is also not clear in this form of writing.

Now as we subtract the 4 tens given away, we move to the second step. Many children take time to learn a two-step division problem; hence we must go very slowly, articulating every action. I usually like to use a downward arrow to indicate "bringing down the next number." This focuses the child's attention on it, makes him understand what is happening, and serves as a visual aid. We now take down 8 units and each gets 2 units which is then recorded on top of 8 as a quotient. After subtraction there are no units left.

ACTIVITY TWELVE

Division of a double digit number by a single digit number (with exchange) and with or without remainder

Materials required: Place value Kit.



Here again division of a double digit number by a single digit number involving exchange is best done through place value material.

Example: Share 64 rupees amongst 4 people.

As shown in the picture, we use tens and units material to show 64 and make 4 groups. Since we need to share 6 tens among 4 people we ask the question: "How many tens can we share out equally among 4 people?" So we first distribute 4 of the 6 tens to the 4 people. (Sometimes children write a lower multiple than what is possible under the dividend and end up with a remainder which is larger than the divisor. Point out to them that when that happens they could have taken a higher multiple). Each gets 1 ten. This is recorded in the division problem as 1 ten in the tens place over the 6. Now when we subtract what is given away to the 4 people, we are left with 2 tens. We now take down 4 units. We convert the 2 tens into 20 units. The number now reads as 24 units. 24 units shared amongst 4 people is 6 units for each person. So we record 6 units over the units place.

Extension: Problems involving remainder should also be taken up and explained using materials in a similar way.

ACTIVITY
THIRTEEN

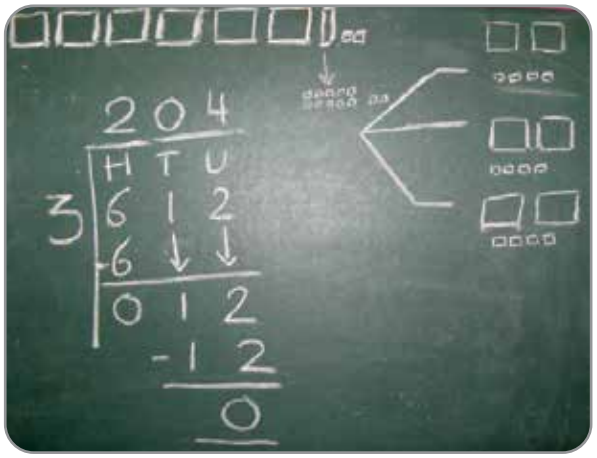
Division of a three digit number by a single digit number (with zero in the quotient)

Materials required: Place value Kit.

Share 612 rupees amongst 3 people:

$$612 \div 3$$

Let us use place value materials to demonstrate this as shown in the picture.

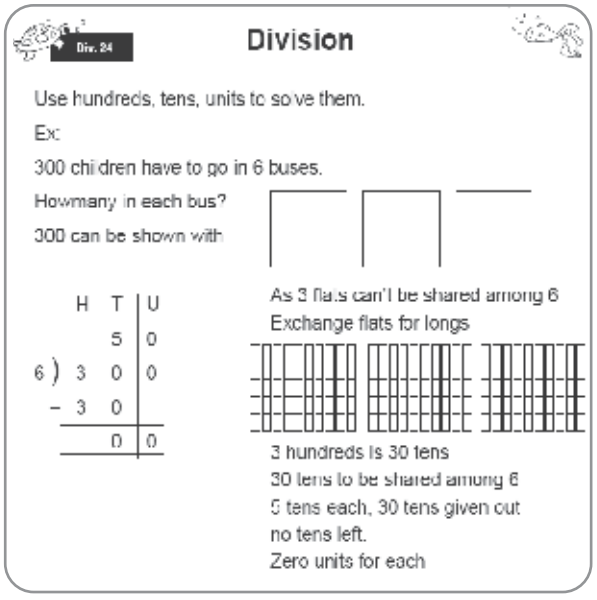


As shown in the picture we use hundreds, tens and units material, or fake money, to show 612. Since we need to share 6 hundreds among 3 people we ask the question: "How many hundreds can we share out equally among 3 people?" So we first distribute 6 hundreds to the 3 people, and each one gets 2 hundreds. This is recorded in the division problem as 2 hundreds in the hundreds place over the 6. Now as we subtract what is given away to the 3 people we are left with 0 hundreds. We now take down 1 ten. We cannot share 1 ten as it is (without exchanging) amongst 3 people. Since no tens are being given out to the three people, we write 0 tens in the tens place of the quotient. Now we bring down the 2 units. We convert the 1 ten into 10 units. The number now reads as 12 units. 12 units shared amongst 3 people is 4 units for each person. Now we record 4 units over the units place.

Extension: Different problems which give rise to zero in the quotient, for example: $408 \div 4$, $400 \div 5$, $600 \div 5$ can be explained using the same kind of reasoning.

While discussing problems which give rise to a remainder we need to draw the child's attention to the fact that the remainder will always be less than the divisor.

Verification: Show the children the method for verifying that their answer is right by multiplying the quotient with the divisor and adding the remainder to the product so obtained. It must match with the dividend.



ACTIVITY FOURTEEN

Division of a double digit number by a double digit number

It is best to postpone teaching this to class 5. Also by this point the child does not (indeed, should not) need to take recourse to the use of concrete materials. Terminology like 'dividend', 'divisor', 'quotient' and 'remainder' can be introduced at this level.

Children face a lot of difficulty in understanding the procedure of division by a double digit number, and many errors happen in this area. Working with multiples of a double digit number is difficult as children do not know the multiplication facts of these numbers. In the initial stages let them construct the multiplication table for the required number and use the facts in solving the problem. At a later point one can help them to use estimation methods to figure out the possible quotient. Estimation would involve looking at the numerals in the highest place value both of the quotient and the divisor. For example, 785 ÷ 24 requires the child to look at 7 hundred and 2 tens which will result in 3 tens (7 hundred divided by 2 tens). Also, let children see that estimation does not always give the exact quotient but it helps in getting close to the possible quotient; it could be at times 1 less than that or 1 more than that.

The games suggested here involve the usage of all four operations. They can be modified to reinforce divisions alone or multiplications and divisions.

GAME 1 CLIMB THE LADDER

Aim: To give practice in using 4 operations with small numbers for two players.

Equipment: Two dice, game board with two ladders (numbers 1 to 10 written on each rung), two counters.

The first player rolls the two dice (let us say 5 and 6 arise), using those numbers and any operation in any combination, he must try to make an answer of 1.

He might arrange them as 6 – 5 = 1. The player can then place his counter on the first rung of the ladder. When a player cannot make the required number, he loses his turn and stays where he is on the ladder.

The second player now rolls the dice in turn to move his counter on the ladder.

Extension: The game can be extended to 20 rungs on the ladder. We can also have 3 dice to bring in 2 operations at a time.

GAME 2 EQUATIONS

Aim: To give practice in using 4 operations with slightly larger numbers.

Equipment: 52 number cards

Numbers 1 to 9 (3 of each), 10 to 18 (2 of each), 19 to 25 (1 of each)

Also make 30 sign cards (+, –, ×, ÷)

No. of players: Two to four

Shuffle the number cards and deal five cards to each player. The object of the game is to arrange these five cards, together with an equals to sign and three sign cards of their choice, to form an equation. Children should be told that × and ÷ get priority over + and –.

A player who manages to make an equation with the original cards scores 10 points. If a player cannot make an equation with his set of 5 number cards and exchanges 1 number card for a new one, then he loses 2 points for every card exchanged.

GAME 3 DOMINOES

Aim: To provide practice in simple multiplication and division at speed.

Equipment: Make a set of 32 dominoes. Each domino is divided by a line into a 'left side' and a 'right side'. The first domino has "Start" written on the left side and a simple problem on the right side (ex. 9 × 3, 24 ÷ 6). On each of the remaining dominoes, the answer for the previous domino's problem is written on the left side, and another problem is given on the right side (the answer for which is on the left side of the next domino, and so on). The last domino has "End" written on the right side.

This game is self-correcting in nature. If a single child plays it alone, he will have dominoes left over if he makes a mistake. If several are playing together, others can point out the mistakes.

Padmapriya Shirali

Padmapriya Shirali is part of the Community Math Centre based in Sahyadri School (Pune) and Rishi Valley (AP), where she has worked since 1983, teaching a variety of subjects – mathematics, computer applications, geography, economics, environmental studies and Telugu. For the past few years she has been involved in teacher outreach work. At present she is working with the SCERT (AP) on curricular reform and primary level math textbooks. In the 1990s, she worked closely with the late Shri P K Srinivasan, famed mathematics educator from Chennai. She was part of the team that created the multigrade elementary learning programme of the Rishi Valley Rural Centre, known as 'School in a Box'. Padmapriya may be contacted at padmapriya.shirali@gmail.com